A REVIEW
OF LINEAR, E x B DRIFT INSTABILITIES

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A REVIEW OF LINEAR, $\underline{E} \times \underline{B}$ DRIFT INSTABILITIES

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ABSTRACT

The linearized Vlasov-Maxwell system of equations has been used extensively to calculate growth rates of instabilities in plasmas supporting particle drifts. This report reviews some recent results in which the basic drift is the electron $\underline{E} \times \underline{B}$ drift but other electron drifts due to plasma and magnetic field inhomogeneities are also present. All gradients are taken in the same direction so the results are of possible application to collisionless shock waves.

1. INTRODUCTION

Drift wave theory /1/ has been used to investigate the linear stability of a large number of plasma configurations. In recent years a particular application has been to plasmas in which the basic drift is due to crossed electric and magnetic fields. Normally, of course, constant electric and magnetic fields give rise to the same drift for both ions and electrons, but there are experimental situations (e.g. collisionless shock waves) in which the electric field is constant over an electron Larmor period but not on the time scale of the much larger ion Larmor period. The net result is that the electrons suffer the usual $\underline{E} \times \underline{B}$ drift but the ions are effectively unmagnetized and only their motion in the direction of the electric field is altered. The same argument applies to the other drifts due to plasma and magnetic field inhomogeneities so that the steady state of the plasma is one in which the electrons have a mean drift relative to the ions and this can serve as an energy source for the growth of unstable waves.

The range of instabilities for the situation just described is considerable. In this review we concentrate on electrostatic waves only, though work on transverse waves has also recently been published /2,3/. Lashmore-Davies and Martin /4/ have given a comprehensive summary of the electrostatic instabilities but allowed for the $\underline{E} \times \underline{B}$ drift only. Although such a model can account for all the instabilities, the introduction of other drifts not only changes the range of unstable wave numbers but can alter growth rates significantly /5-10/. However, the most serious criticism of a treatment based on the $\underline{E} \times \underline{B}$ drift alone is that it is a self-consistent model only in the limit of vanishing β_e; as the work of Gary, Sanderson and Priest /9-14/ has shown, even very small but finite β_e can lead to a drastic reduction in the growth rate of some instabilities.

In this review our aim is to give a physical description of the basic $\underline{E} \times \underline{B}$ drift instabilities, including their limiting conditions, and to point out the modifications arising from finite β_e and pressure gradient effects. Very little mathematical detail is given since this is to be found in the original papers.

2. STEADY STATE MODEL OF THE PLASMA

The usual model for the steady state plasma is that shown in figure 1. If the ion and electron Larmor radii are ρ_i and ρ_e, respectively, the scale length L of the inhomogeneities must be such that

$$\rho_e << L << \rho_i \tag{1}$$

in order that the ions be treated as unmagnetized. Furthermore, since drift wave theory requires weak gradients

$$\varepsilon\rho_e, \varepsilon'\rho_e, \delta\rho_e << 1 \tag{2}$$

or, equivalently, the corresponding drift velocities ($\bar{v}_B = \varepsilon v_e^2/\Omega_e$, $v_n = \varepsilon' v_e^2/\Omega_e$, $v_T = \delta v_e^2/\Omega_e$) must be small compared with the electron thermal velocity, v_e; Ω_e is the electron cyclotron frequency. Finally, the local approximation /1/ requires

$$\varepsilon\lambda, \varepsilon'\lambda, \delta\lambda << 1 \tag{3}$$

where λ is the wavelength of whatever waves we may be considering.

For the electrons the steady state equilibrium is maintained by the balance of the electrostatic force and the opposing pressure gradient, $q\underline{E} = \underline{\nabla}(p + B^2/8\pi)$, giving, in terms of the drift velocities,

$$v_0 = v_n + v_T + \frac{2\bar{v}_B}{\beta_e}$$

where $v_0 = cE_0/B$ is the $\underline{E} \times \underline{B}$ drift.

The net drift velocity v_d is determined by the magnetic field gradient via Ampère's law, $\underline{j} = c\underline{\nabla} \times \underline{B}/4\pi$, that is

$$v_d = \frac{2\bar{v}_B}{\beta_e}. \tag{4}$$

Hence,

$$v_d = v_0 - v_n - v_T. \qquad (5)$$

Thus, the macroscopic drift velocity is the difference between the $\underline{E} \times \underline{B}$ drift and the sum of the $\underline{\nabla}n$ and $\underline{\nabla}T$ drifts /9/.

If one looks at the plasma miscroscopically the motion of an electron shows a drift which is the resultant of v_0 and $v_B = \varepsilon v_\perp^2/2\Omega_e$, where $v_\perp$ is the electron's velocity perpendicular to the magnetic field. In taking the average over all electrons in a fluid element the $\underline{\nabla}B$ drift makes only a second order contribution to the net drift velocity and does not therefore appear on the right-hand side of equation (5). One cannot conclude from this, however, that the effects of the $\underline{\nabla}B$ drift are negligible. Although $v_B/v_0 \sim \beta_e$, the microscopic role of the $\underline{\nabla}B$ drift may be crucial even for low β_e plasmas as we shall discuss in the next section; the essential point here is that the velocity dependence of v_B destroys the uniformity (all electrons drifting with v_0) which would otherwise exist.

Returning to the macroscopic picture, the drifts v_n and v_T appear only on taking the average over all the electrons in a given plasma sheet, as shown schematically in figure 2. Electrons with guiding centres to the right of the sheet contribute negatively to the velocity average and, in the case of a density gradient, there are more of them than electrons with guiding centres to the left, which contribute positively. A similar argument applies in the case of a temperature gradient and shows why v_d is the difference between v_0 and $v_n + v_T$, as given by equation (5).

For the ions there is no equilibrium but a simple Maxwellian distribution is usually assumed. Thus in the absence of plasma inhomogeneities the steady state distribution functions are Maxwellians displaced from each other by the drift velocity $v_d = v_0$. It is easy to show /9/, under the weak gradient approximation (2), that the introduction of a density gradient does not change the shape of the electron distribution but merely shifts it by an amount v_n such that now $v_d = v_0 - v_n$; or, if we regard the magnetic field gradient as constant, then v_d is also constant by equation (4), and the introduction of a density gradient means that v_0 must increase by v_n.

In contrast with this, the introduction of an electron temperature gradient leads to a distortion of the distribution function as well as an increase in v_0 (assuming v_d is held constant) /9/. This is illustrated in figure 3 which is drawn in the average rest frame of the electrons. Again we consider a plasma sheet and it is easily seen that electrons with guiding centres to the right of the sheet contribute to the left (negative v_y) side of the distribution function and vice versa. When the temperature increases with x the spread in velocities to the right of the sheet is greater than to the left with the resultant distortion of $f_e^{(0)}$. The displaced and distorted electron distribution function is shown in figure 4; note that the peak has moved from v_d to $v_d + \frac{3}{2}v_T$. We shall see that both the distortion and the shift of the peak may be of considerable significance.

One of the chief applications of this model has been to collisionless shocks. For such applications one should, of course, include an additional electric field component $E_y = u_s B_0/c$, where u_s is the shock velocity. In the absence of density and temperature gradients the effect of E_y is merely to Doppler shift the wave frequency by an amount $\underline{k}.\underline{u}_s$ /15/. In general, however, the flow of the plasma into the front of the shock means that the description given by figures 2 and 3 is over-simplified, the true drift being at an acute angle to the gradients.

Finally, we note that from equations (4) and (5) and the definitions of the drift velocities it follows that in the $\beta_e = 0$ limit we may choose

$$v_d = v_0 > 0 \quad , \quad v_T = v_n = v_B = 0. \tag{6}$$

On the other hand, for finite β_e, although we may choose $v_T = v_n = 0$, v_B must be non-zero for a self-consistent model.

In the following section, it is the limit of vanishing β_e that we consider first for each instability and then investigate the modifications arising from the effects of finite β_e and plasma inhomogeneities.

3. ELECTROSTATIC INSTABILITIES

The $\beta_e = 0$ approximation has been investigated by many authors. The two waves which are of interest in this situation are the Bernstein modes and ion acoustic waves /11/. The properties of Bernstein waves /16/ depend entirely on the electrons and because the electrons are restricted to circular motion perpendicular to the magnetic field the waves propagate without damping in this plane. On the other hand, there is no such restriction on electron motion along the field so that whenever the wave vector has a component, $k_\parallel = k \cos\theta$, along $\underline{B}$, Landau damping can occur /17a/. Damping will be severe if $\omega/k_\parallel \lesssim v_e$ and it follows that the *nth* harmonic ($\omega \sim n\Omega_e$) propagates only at angles such that

$$\cos\theta \ll \frac{n}{k\rho_e}. \tag{7}$$

Ion acoustic waves involve both ions and electrons; the ions provide the inertia but the more mobile electrons are responsible for the propagation of the waves /17b/. In this case the restriction of the electrons (to circular motion) in the plane perpendicular to $\underline{B}$ means that ion acoustic waves do not propagate in this plane at wavelengths $\lambda \gtrsim \rho_e$. The freedom of electron motion parallel to the magnetic field means that ion acoustic waves can propagate with a non-zero $k_\parallel$; the condition that the electrons be sufficiently mobile for propagation of the waves is $\omega/k_\parallel \ll v_e$. A second limit on $\omega/k_\parallel$ is provided by the need to avoid ion Landau damping; this condition is $v_i \ll \omega/k_\parallel$. Writing $\omega \sim kc_s$, where

$$c_s = \left[\frac{T_e}{m_i}\left(\frac{3T_i}{T_e} + \frac{1}{1 + k^2\lambda_D^2}\right)\right]^{\frac{1}{2}}$$ is the ion acoustic phase velocity, and

rearranging these limits we get

$$\left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} << \cos\theta << \begin{cases} (T_e/T_i)^{\frac{1}{2}} & (T_e >> T_i) \\ 1 & (T_e \lesssim T_i) \end{cases} \qquad (8)$$

Thus, for $k\rho_e >> 1$ ion acoustic waves behave as in the zero magnetic field case and are ion Landau damped for $T_e \sim T_i$ /12/; for $k\rho_e \lesssim 1$ ion acoustic waves propagate at angles satisfying (8), which may, of course, include propagation at $T_e \lesssim T_i$ /18,19/. Let us note, too, that in a magnetized plasma the ion acoustic wave frequency tends to the lower hybrid frequency as $k \to 0$, unlike the $\underline{B} = 0$ case where it tends to zero.

3.1 Ion acoustic instability

In discussing the instabilities which arise from these waves it is convenient to begin with ion acoustic waves in the "non-magnetic" case $k\rho_e >> 1$ and propagating at an acute angle to the magnetic field. As is well known, a drift velocity v_d such that

$$v_i << c_s < v_d \qquad (9)$$

makes ion acoustic waves unstable; the instability arises from inverse electron Landau damping and the growth rate is proportional to $(\underline{k}.\partial f_e^{(0)}/\partial \underline{v})$ evaluated at $\underline{k}.\underline{v} = \omega$. The first inequality in (9) may be satisfied for $T_i << T_e$ but not at equal temperatures.

The $\underline{\nabla B}$ drift (finite β_e) has very little effect on this instability /9,12/. This is not surprising since in this effectively non-magnetic regime the electrons are thermal in all directions and since $k_\parallel > 0$ a spread in the resonant phase is already present $(k_\parallel v_\parallel)$. Introducing a spread in $v_\perp$ space by the small term $k_y v_B$ produces only a marginal reduction in the growth rate, therefore (unless, of course, $k_\parallel$ becomes very small compared with k_y; see below. A ∇n drift, too, has only a very small effect associated with the increase in v_0 /9/.

However, we have seen already that a temperature gradient distorts the distribution function and in the region of unstable waves this represents an increase in the positive slope (see figure 4). The increase in the growth rate is equal to this increase in slope ($\gamma(\nabla T)/\gamma(0) = 1 + 3v_T/2v_d$) and, in fact, one can obtain the correct result simply by substituting the formula for the distorted distribution in the usual $\underline{B} = 0$ expression for the ion acoustic growth rate /9/.

3.2 Electron cyclotron instability

For $k\rho_e >> 1$ ion acoustic waves continue to propagate as θ approaches $\pi/2$. However, at right angles to the magnetic field, or, more precisely, at angles satisfying (7), Bernstein waves propagate and couple with the ion acoustic waves to give a new instability /11, 20, 21/, known variously as the electron cyclotron, beam cyclotron or Bernstein instability. Following reference 4, we shall use the first of these names for this reactive instability, reserving the last name for the dissipative instability involving Bernstein waves which is discussed in the next sub-section.

One expects /4,22/ that the change from a dissipative instability (involving just a few resonant electrons) to a reactive one (involving all the electrons in a wave-wave coupling) would lead to an increased growth as θ tends to $\pi/2$ and, indeed, this is so, the maximum growth taking place at $\theta = \pi/2$ /12/. Of course, both instabilities disappear as T_i approaches T_e because of Landau damping of the ion acoustic waves.

With $k_\parallel \approx 0$, the $\nabla\underline{B}$ drift now has a very marked stabilizing influence since it introduces a spread in what is otherwise a very sharp resonance. At $\beta_e = 0$, all electrons drift with the same microscopic drift velocity v_0 and, therefore, all participate in the resonance. By including v_B this sharp resonance is smeared out; the greater an electron's perpendicular velocity the less effectively it participates in the reactive coupling. A point to note here is that the higher harmonics involve electrons with higher velocities and therefore suffer greater reductions in the instability growth rate. The size of the reduction will depend in some way on the ratio of the spread in the resonant frequency $\Delta\omega \sim \gamma$ to $k\bar{v}_B$. For the electron cyclotron instability /10/

$$\frac{\gamma(\nabla B)}{\gamma(0)} \sim \left(\frac{\gamma}{k\bar{v}_B}\right)^{1/3}$$

Since $\gamma \sim n^{\frac{1}{2}}$ and $k \sim n$, where n is the mode number, this ratio varies with n only as $n^{-\frac{1}{6}}$ so that the reduction does not depend strongly on mode number, as shown in Gary's numerical results /13/.

For small β_e, plasma inhomogeneities have only a marginal effect on this instability /5,10,23/. Reactive instabilities are insensitive to changes in $f_e^{(0)}$. One interesting effect is the increase in the range of unstable phase velocities brought about by the distortion of the distribution function; this becomes $v_i < v_{ph} < v_d + \frac{3}{2}v_T$ instead of (9) and some previously stable modes are then unstable /10/.

3.3 Bernstein instability

Although both the instabilities we have discussed so far disappear as T_i approaches T_e, two new instabilities arise to take their place. The first of these also involves Bernstein waves and appears therefore only at propagation angles satisfying (7). This Bernstein instability /24-26/ is a dissipative one arising because, in the rest frame of the electrons, the electron Bernstein modes see a positive slope to the ion distribution function and can therefore undergo inverse ion Landau damping. The maximum growth rate occurs at the point of maximum slope of the ion distribution function and is typically of the same order of magnitude as the ion acoustic growth rate.

What was said in the last sub-section concerning finite β_e effects applies equally well here except that the reduction in the growth rate is now strongly dependent on the mode number. For the Bernstein instability

$$\frac{\gamma(\nabla B)}{\gamma(0)} \sim \frac{\gamma}{k\bar{v}_B}$$

and, since in this case $\gamma(0)$ is not dependent on n, the reduction factor varies as n^{-1}. Furthermore the linear dependence on $\gamma/k\bar{v}_B$ is much stronger than the one third power dependence for the electron cyclotron instability. Thus, for a given value of β_e, the percentage reduction in the Bernstein instability is greater than for the electron cyclotron instability and the n = 1 mode is singled out as the most unstable /10,13/. The *nth* mode is stabilized for $n\beta_e > 2(v_0 - v_i\sqrt{2})/v_d$.

The same remarks regarding the effect of plasma inhomogeneities on the range of unstable phase velocities apply to the Bernstein instability as to the electron cyclotron instability. However, in this case $\gamma(0) \sim (v_d - v_i\sqrt{2})$ and on introducing a pressure gradient this becomes $\gamma(\nabla p_e) \sim (v_d + \frac{3}{2}v_T - v_i\sqrt{2})$ owing to the shift of the peak of the electron distribution function. Clearly, this can represent a considerable increase in growth rate /8/.

3.4 Modified two-stream instability

The last instability we must consider is related to the ion acoustic instability but in the long wavelength limit, $k\rho_e \lesssim 1$. As explained at the beginning of this section, for waves with $\lambda \gtrsim \rho_e$ the electrons are effectively cold in the perpendicular directions and thermal only along the field. The growth rate for the ion acoustic instability varies as the slope of the electron distribution function in the direction of propagation of the wave. In this direction the distribution appears as a combination of components of a Maxwellian parallel to $\underline{B}$ and a delta function perpendicular to $\underline{B}$. The net result is a Maxwellian with a reduced thermal velocity $\bar{v}_e = v_e \cos\theta$ in the $\underline{k}$

direction /19/. Since the growth rate $\gamma \propto \underline{k}.\partial f_e^{(0)}/\partial \underline{v} \sim k/v_e \cos\theta$, this now shows an increase over the $\underline{B} = 0$ case by the factor $(\cos\theta)^{-1}$. (Since $\cos\theta$ must satisfy (8) and $k < \rho_e^{-1}$ this "magnetic" ion acoustic instability is insignificant compared with the $k >> \rho_e^{-1}$ "non-magnetic" ion acoustic instability for $T_i << T_e$.)

Now let us consider what happens as we reduce $\cos\theta$ to $\sim (m_e/m_i)^{\frac{1}{2}}$. The ion acoustic wave can no longer propagate because of the reduced thermal velocity. However, the ion acoustic wave goes into the lower hybrid wave in the presence of a magnetic field as $k \to 0$, and the dissipative ion acoustic instability gives way to a reactive instability, again with an increase in growth rate /4,6,18,19/. The lower hybrid wave couples with an electron mode whose frequency is either $\Omega_e \cos\theta\,(\omega_{pe} << \Omega_e)$ or $\omega_{pe} \cos\theta\,(\omega_{pe} << \Omega_e)$. Even though the drift velocity $v_d << v_e$ the reduction in the effective thermal velocity creates a situation analogous to the usual two-stream instability (see figure 5) and hence the name. For the $\underline{B} = 0$ ion acoustic instability, as T_i increases to T_e, instability can only be maintained by increasing v_d to v_e, at which point one has the two-stream instability. In a magnetized plasma, as $T_i \to T_e$, one can maintain instability by reducing $\cos\theta$ so that the effective thermal velocity $\overline{v}_e$ reduces to v_d, thus giving the modified two-stream instability.

Since $k_\parallel/k << 1$ the spread in the resonance due to the integration over $v_\parallel$ is not as significant as in the case of the ion acoustic instability (which is why this is a reactive rather than a dissipative instability). Introducing the ∇B drift, therefore, should be more effective in reducing the growth rate than for the ion acoustic instability. This conjecture is borne out by Gary's numerical results /14/.

Since the modified two-stream instability does not depend critically on the shape of the distribution function pressure gradient effects are not crucial /6/. However, one interesting result of introducing plasma or field inhomogeneity is that the waves then couple at $k_{\parallel} = 0$ which is not otherwise so /4/.

Table 1 summarizes the instabilities together with the conditions for their existence and their responses to $\underline{\nabla}B$ and $\underline{\nabla}p_e$ drifts.

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References

1. N. A. Krall, "Drift Waves" in Advances in Plasma Physics, Vol.1 (eds. A. Simon and W. B. Thompson) Interscience Publishers, N.Y. (1968).
2. C. S. Wu, J. Geophys. Res. 76, 4454 and 6961 (1971).
3. M. Lampe, W. M. Manheimer, J. B. McBride, J. H. Orens, K. Papadopoulos, R. Shanny, and R. N. Sudan, Phys. Fluids, 15, 662 (1972).
4. C. N. Lashmore-Davies and T. J. Martin, 2nd Topical Conf. on Pulsed High-Beta Plasmas, Garching, Paper D5 (1972).
5. C. N. Lashmore-Davies, Phys. Fluids, 14, 1481 (1971).
6. N. A. Krall and P. C. Liewer, Phys. Rev. A4, 2094 (1971).
7. J. J. Sanderson and E. R. Priest, Proc. Xth International Conf. on Phenomena in Ionized Gases, Oxford, p.66 (1971).

8. J. J. Sanderson and E. R. Priest, J. Phys. A: Gen. Phys. 4, L65 (1971).

9. E. R. Priest and J. J. Sanderson, Plasma Phys. (to be published) (1972).

10. J. J. Sanderson and E. R. Priest, Plasma Phys. (to be published) (1972).

11. S. P. Gary and J. J. Sanderson, J. Plasma Phys. 4, 739 (1970).

12. S. P. Gary, J. Plasma Phys. 4, 753 (1970).

13. S. P. Gary, J. Plasma Phys. (to be published) (1972).

14. S. P. Gary, "Long Wavelength Instability in a Perpendicular Shock" (1972).

15. S. P. Gary and J. J. Sanderson, Proc. Vth Yugoslav Symposium and Summer School on the Physics of Ionized Gases, Hercegnovi, p.147 (1970).

16. I. B. Bernstein, Phys. Rev. 109, 10 (1958).

17. P. C. Clemmov and J. P. Dougherty, "Electrodynamics of Particles and Plasmas", Addison-Wesley, Reading, Mass (a) p.290, (b) p.274 (1969).

18. V. I. Aref'ev, Sov. Phys. Tech. Phys. 14, 1487 (1970).

19. D. J. Barrett, B. D. Fried, C. F. Kennel, J. M. Sellen and R. J. Tayler Phys. Rev. Letters 28, 337 (1972).

20. H. V. Wong, Phys. Fluids 13, 757 (1970).

21. M. Lampe, J. B. McBride, J. H. Orens and R. N. Sudan, Phys. Letters 35A, 131 (1971).

22. A. Hasegawa, Phys. Rev. 169, 204 (1968).

23. J. J. Sanderson and S. P. Gary, Proc. IVth European Conf. on Controlled Fusion and Plasma Physics, Rome, p.54 (1970).

24. C. N. Lashmore-Davies, J. Phys. A3, L40 (1970).

25. D. W. Forsland, R. L. Morse and C. W. Nielson, Phys. Rev. Letters 25, 1266 (1970).

26. S. P. Gary and D. Biskamp, J. Phys. A: Gen. Phys. 4, L27 (1971).

27. K. N. Stepanov, Sov. Phys. Tech. Phys. 9, 1653 (1965).

28. D.E.T.F. Ashby and A. Paton, Plasma Phys. 9, 359 (1967).

Table and figure captions:

Table 1: Electrostatic instabilities with frequency and growth rate much greater than ion Larmor frequency in $T_i \lesssim T_e$ plasmas. All drifts are in the y direction, $\underline{B} = (0,0,B)$ and $\underline{k} = (0, k \sin\theta, k \cos\theta)$.

Figure 1: Steady state plasma model.

Figure 2: Macroscopic drift velocity in inhomogeneous plasma. The microscopic drift velocity of all electrons is v_0 but on taking the average velocity in a plasma sheet at x_0 there are more negative contributions from the greater number of electrons to the right of the sheet than positive contributions from the left.

Figure 3: Schematic representation of ∇T distortion of electron distribution function. Electrons with guiding centres at $x < x_0$ contribute to $f_e^{(0)}$ at $v_y > 0$ and vice versa. A temperature gradient decreases the spread at $x < x_0$ and increases it at $x > x_0$.

Figure 4: Temperature gradient distortion gives (i) increased slope of $f_e^{(0)}$ at unstable phase velocities (ii) increased range of unstable phase velocities.

Figure 5: Effective velocity distributions in direction of wave propagation for $k\rho_e \lesssim 1$ and $\cos\theta \sim (m_e/m_i)^{\frac{1}{2}}$. The electron distribution is the resultant of a small component of the Maxwellian at $\theta = 0$ and a large component of a delta function (effectively cold electrons) at $\theta = \pi/2$.

Instability	Type	Conditions	Frequencies	Growth Rate varies as	Response to ∇B (finite β_e)	Response to $\nabla p = \nabla n + \nabla T$
Electron Cyclotron	Reactive	(i) $k\rho_e >> 1$ (ii) $n >> k\rho_e \cos\theta$ (iii) $T_e >> T_i$	$k\, c_s \approx k\, v_0 - n\Omega_e$	$\sqrt{n}$ (for small n)	moderate reduction, weakly dependent on n	weak
Ion Acoustic	Dissipative	(i) $k\rho_e >> 1$ (ii) $v_d > c_s$ (iii) $T_e >> T_i$	$k\, c_s$	$\left(\underline{k}.\frac{\partial f_e^{(0)}}{\partial \underline{v}}\right)_{v_{ph}}$	weak	$\frac{\gamma(\nabla T)}{\gamma(0)} \sim 1 + \frac{3v_T}{2v_d}$ (ii) becomes $v_d + \frac{3}{2}v_T > c_s$
Bernstein	Dissipative	(i) $n >> k\rho_e \cos\theta$	$k\, v_i\sqrt{2} \approx k\, v_0 - n\Omega_e$	$(v_d - v_i\sqrt{2})$ $(k\rho_e >> 1)$	strong reduction, increasing with n, stabilization for $n\beta_e v_d > 2(v_0 - v_i\sqrt{2})$ $(k\rho_e >> 1)$	$\frac{\gamma(\nabla T)}{\gamma(0)} \sim 1 + \frac{3v_T}{2v_d}$ $(k\rho_e >> 1)$
Modified Two-stream	Reactive	(i) $k\rho_e \lesssim 1$ (ii) $0 < \cos\theta \sim \sqrt{\frac{m_e}{m_i}}$	$\omega_{LH} \approx k\, v_0 \sin\theta - \omega_{LH}\cos\theta\sqrt{\frac{m_i}{m_e}}$	ω_{LH}	moderate (ii) becomes $0 \le \cos\theta \sim \sqrt{\frac{m_e}{m_i}}$	weak (ii) becomes $0 \le \cos\theta \sim \sqrt{\frac{m_e}{m_i}}$

Table 1

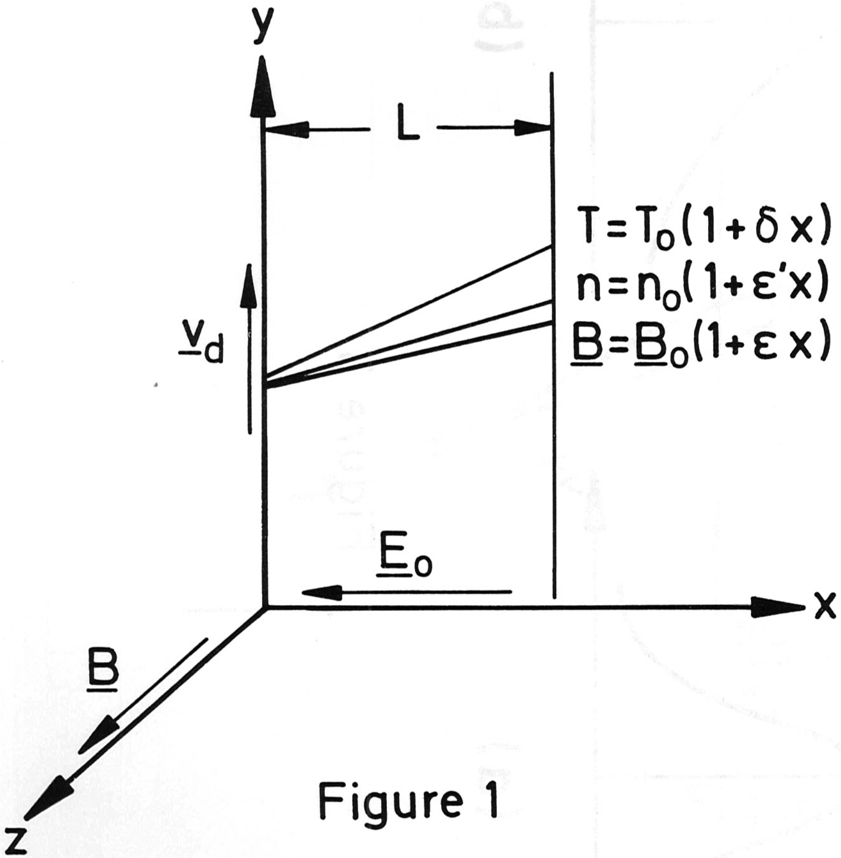

Figure 1

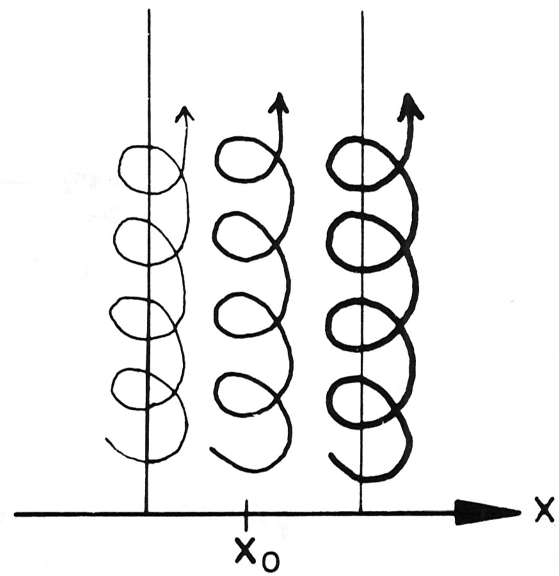

Figure 2

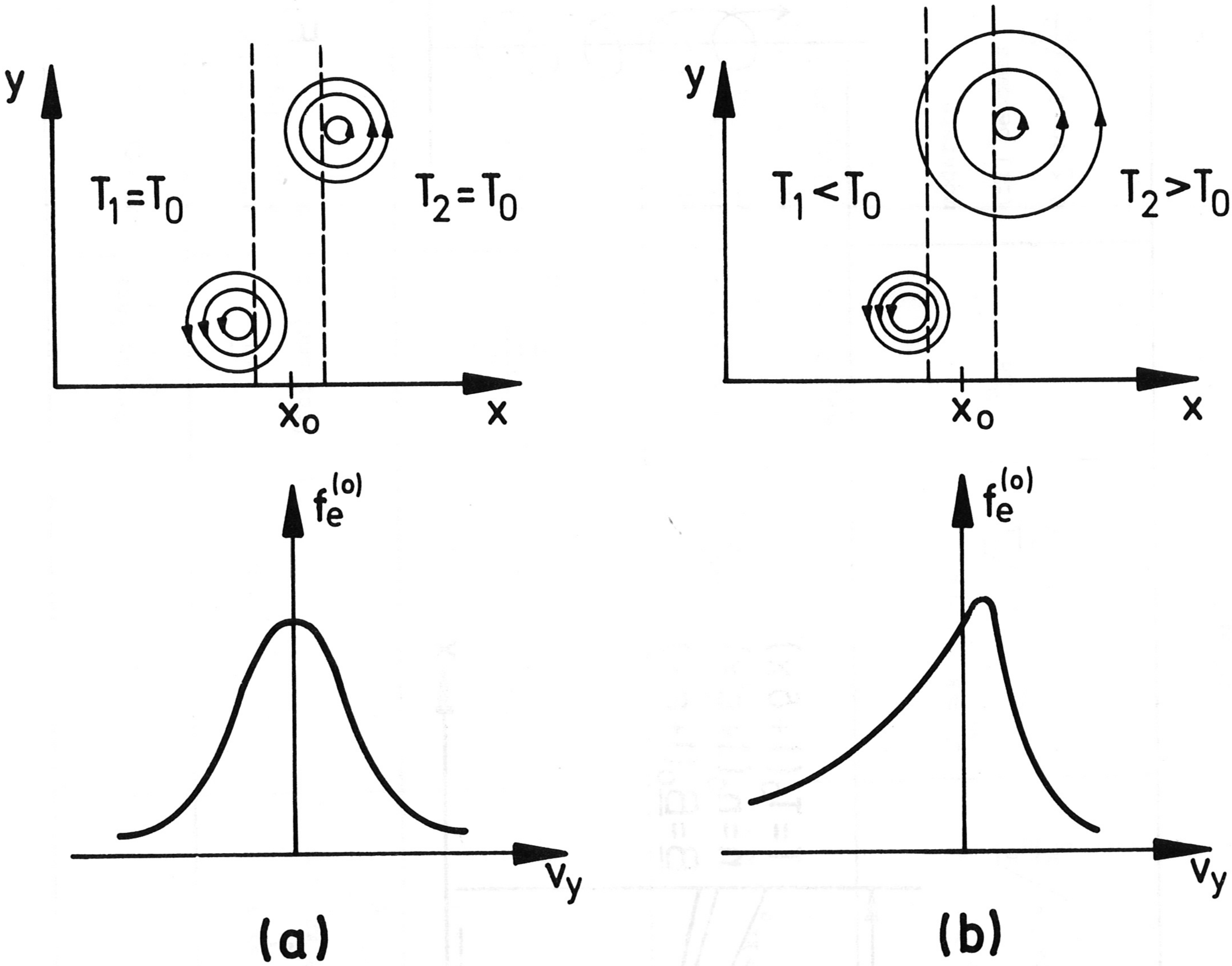

Figure 3

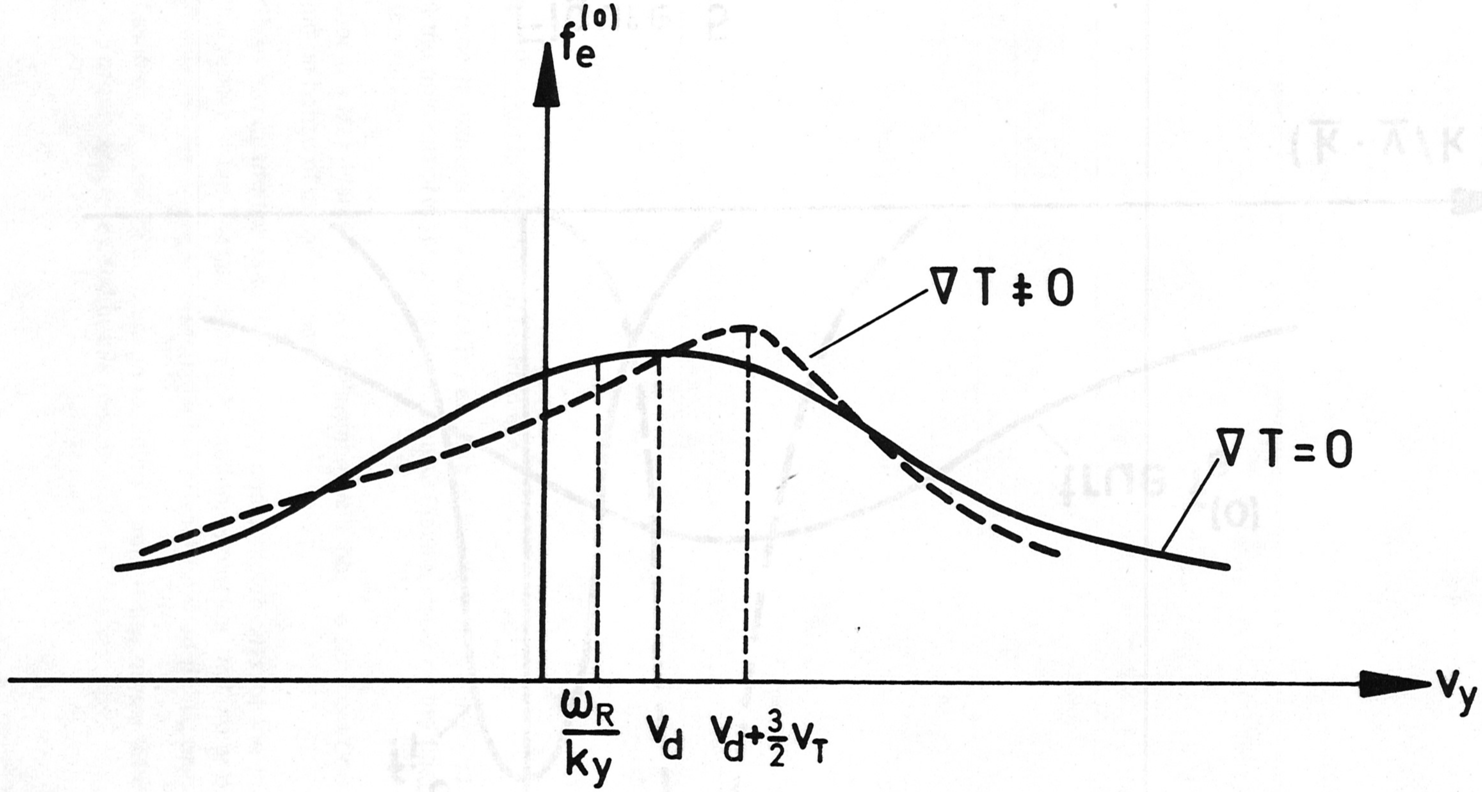

Figure 4

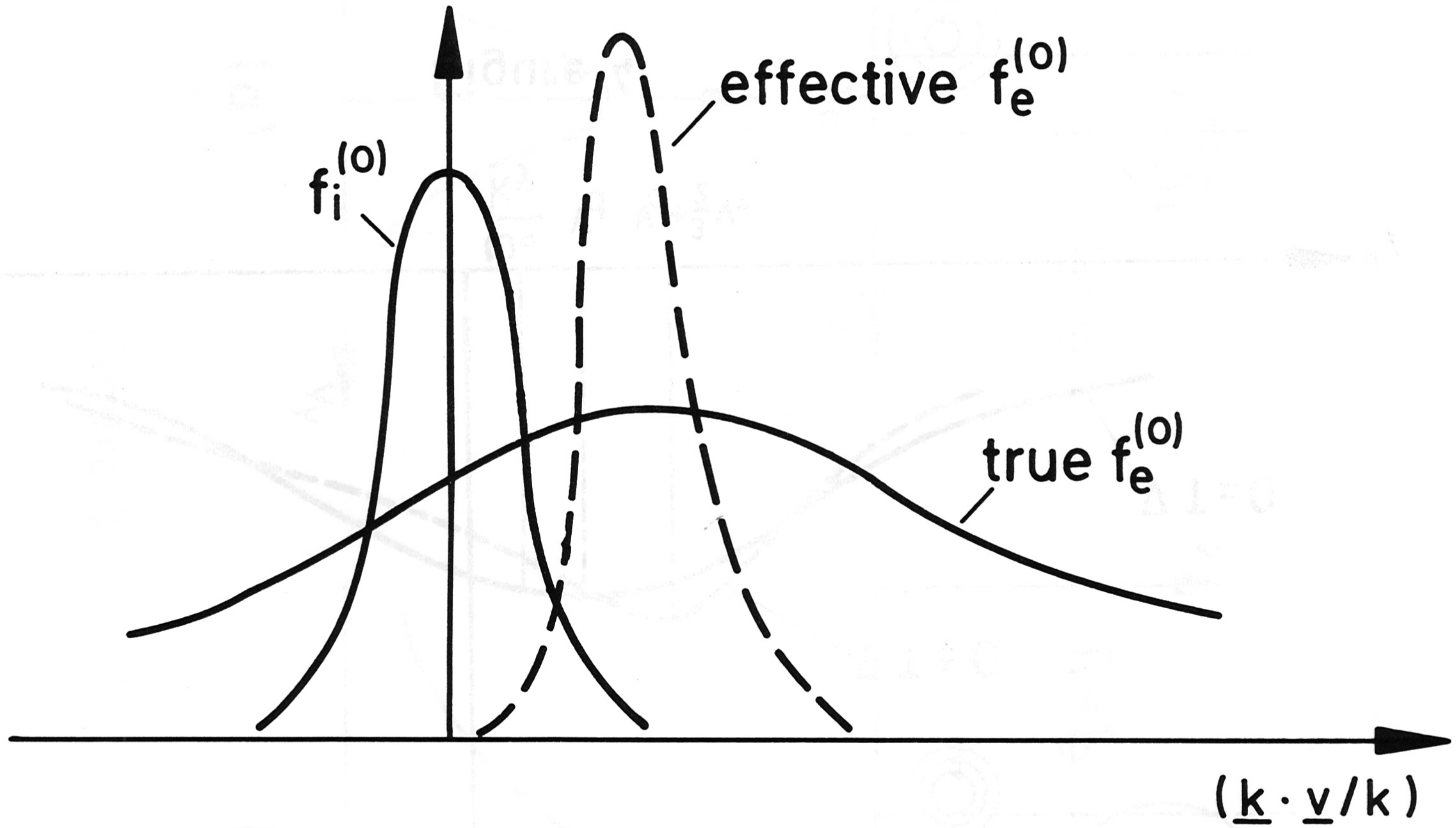

Figure 5